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On the Dynamical System Models to General Transmission Systems with Nonlinear Coupling Noises

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Synopsis. The transmission systems with nonlinear coupling noises (non-additive noises) and additive noises are treated here. Particularly, the nonlinear coupling noise is very interesting and important. For example, it is appeared in optical fiber communication systems and is made from the properties of APD (Avalanche Photo Diode). This paper shows the dynamical system models to general transmission systems with nonlinear coupling noises, for the reason that such models are not treated yet. The dynamical system models obtained here ere very useful, for they can yield the estimation, control and identificat ion problems to the transmission systems. Finally, an illustrative example is given.

1. Introduction

The importance of optical trnansmission prodlems is now increasing. Transmitted signals through the transmission lines (channels) are usually randomly distorted. Received signal is generally thought of a noisy transformed signal and it has generally not only an additive noise component such as thermal noise but also non-additive noise components as nonlinear coupling noises. Authors(1) have already discussed such nonlinear coupling noise problems. Then it is already known that multiplicative noise is appeared in the optical fiber transmission line under the some conditions(2). Such a non-additive noise is very interesting and important in communication and modern control systems.

In the transmission systems, almost systems are scalar: each input and output signal is scalar. Hence, if we intend to treat filtering ploblems to the transmission systems, Wiener filter theory(3)(stationary problems)or extended Wiener filter theory(4) (nonstationary problems) can be applied. But in their filter approaches, it is very hard analytically or numerically to obtain the solution of filters to the non-additive noises: the filter has to be nonlinear structure and then mathematical treatment becomes complicated and requires much efforts to obtain the solution of filters. And so modern filter theory (state estimation approach) is available. We must now note the fact that the applications of the modern filter theory to the transmission systems including the non-additive noises such as nonlinear coupling noises are not treated yet. However if we adopt modern filter theory, the dynamical systems (state systems) to the transmission systems should be first required to obtain.

The purpose of this paper is to show the dynamical system models to the transmission systems. Where real systems are only treated here. Such models are very useful because they induce the application to the problems of estimation, control and identification prodlems. In addition, an illustrative example is shown.

2. Dynamical system models

We denote transmission system model in Fig. 1 where input or output signals are vectors and they are defined as follows. State N_a-vector $a(t)$ means transmitted signal and is governed dy the following dynamical system[6]. However, in order to simplfy the discussions, we assume through the paper that the transmitted signal is no modulated. It must de noted that this assumption is allowable, for example, in optical fiber transmission systems[2].

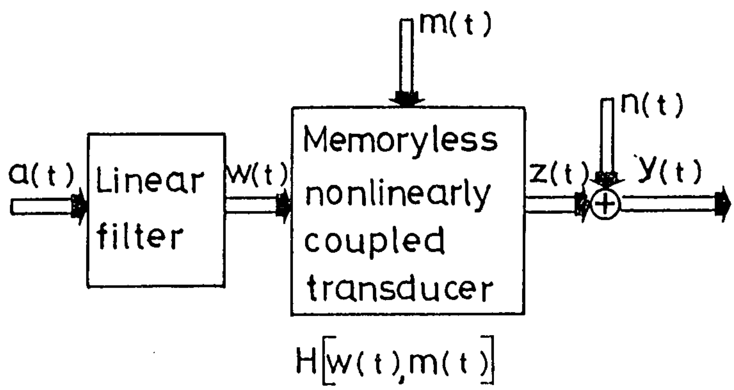

Fig. 1 General transmission system model

$$\dot{a}=f_a[t,\ a(t)]+g_a(t)u_a(t) \tag{1.1}$$

$$a(t_0)=a_0 \tag{1.2}$$

Where symbol • denotes the first time derivative and t_0 is initial time. Then M_a-vector $u_a(t)$ (all vectors are defined as column vectors) denotes white gaussian system noise with zero mean and it's covariance is defined dy the following equation.

$$E[u_a(t)u_a^T(\tau)]=Q^{(a)}(t)\delta(t-\tau) \tag{2}$$

where E means expectation operator, T is transpose and $\delta(t)$ is Dirac delta function. For such a gaussian noise $u_a(t)$, we adopt, subsequently, the expression $u_a(t)\sim N\ [O,\ Q^{(a)}(t)]$, where O is zero vector and matrix $Q^{(a)}(t)$ is time continuous and symmetric nonnegative definite. Vector value functional $f_a[t,\ a(t)]$ ane also matrix function $g_a(t)$ is time continuous. Furthermore each of them has fitted size. Then it is supposed that a_0 is gaussian stochastic vector with mean $\bar{a}_0$ and covariance matrix $p_0^{(a)}$ where

$$\bar{a}_0=E[a_0] \tag{3.1}$$

$$p_0^{(a)}=[(a_0-\bar{a}_0)\ (a_0-\bar{a}_0)^T] \tag{3.2}$$

morever a_0 and $u_a(t)$ are assumed uncorrelated.

Almost transmission systems have two part of transformation, as given in Fig. 1, to the transmitted signal: first, the filter which has input vector $a(t)$ and output vector $w(t)$, secondly, memoryless nonlinear transducer with input vectors $w(t)$, $m(t)$ and output vector $z(t)$. Where $m(t)$ expresses random disturbance vector and besides means exciting noise input which makes nonlinear distortions. Then $z(t)$ it said to be the signal dependent on $a(t)$. We call here $m(t)$ nonlinear coupling noise. And it should be noted that typical additive noise $n(t)$ is thermal noise. Thus received signal vector $y(t)$ through the transmission lines is the sum of $z(t)$ and $n(t)$.

Now linear filter is defined as follows.

$$\dot{\alpha}(t)=F_{\alpha}(t)\alpha(t)+G_{\alpha}(t)a(t) \quad (4.1)$$

$$\alpha(t)=\alpha_0 \quad (4.2)$$

$$w(t)=F_w(t)\alpha(t)+G_w(t)a(t) \quad (4.3)$$

where N_α-vector $\alpha(t)$ is state vector and observed signal N_w-vector $w(t)$ is the output of the linear filter. Matrices $F_a(t)$, $G_\alpha(t)$, $F_w(t)$ and $G_w(t)$ are assumed time continuous and well fitted size. N_α—vector α_0 is initial state vector with mean $\bar{\alpha}_0$.

The dynamical system to nonlinear coupling noise $m(t)$ is modeled a follows.

$$\dot{\beta}(t)=f_\beta[t,\ \beta(t)]+g_\beta(t)u_\beta(t) \quad (5.1)$$

$$\beta(t_0)=\beta_0 \quad (5.3)$$

$$m(t)=f_m[t,\ \beta(t)] \quad (5.3)$$

where N_β-vector $\beta(t)$ denotes state vector and M_β-vector $u_\beta(t)\sim N[O,\ Q^{(\beta)}(t)]$ means system noise and white. $M_\beta\times M_\beta$ matrix $Q^{(\beta)}(t)$ is time continuous and besides symmetric nonnegative defnite. Thus nonlinear couplig noise N_m-vector $m(t)$ is supposed given by the observation process (5.3). Where $f_\beta[t,\ \beta(t)]$ and $f_m[t,\ m(t)]$ are vector value functionals which are time continuous. Matrix $g_\beta(t)$ is also assumed time continuous. Of course they are supposed fitted size. Initial state N_β-vector β_0 is gaussian stochastic constant vector with mean $\bar{\beta}_0$ and covariance $p_0^{(\beta)}$ where

$$\bar{\beta}_0=E[\beta_0] \quad (6.1)$$

$$p_0^{(\beta)}=E[(\beta_0-\bar{\beta}_0)\ (\beta_0-\bar{\beta}_0)^T] \quad (6.2)$$

furthermore β_0 and $u_\beta(t)$ are uncorrelated. Before constructing the model to the additive noise, we must take a consideration that the additive noise can not be practically white noise. For that reason, it is supposed the following dynamical system.

$$\dot{\theta}(t)=f_\theta[t,\ \theta(t)]+g_\theta(t)u_\theta(t) \quad (7.1)$$

$$\theta(t_0)=\theta_0 \quad (7.2)$$

$$n(t)=f_n[t,\ \theta(t)]+\epsilon_n v_n(t) \quad (7.3)$$

where N_θ-vector $\theta(t)$ denotes state vector and system noise M^θ-vector $u_\theta(t)\sim N[O, Q^{(\theta)}$ (t) is white noise. $M_\theta\times M_\theta$ matix $Q^{(\theta)}(t)$ is time continuous and symmetric non-negative definite. Thus additive noise $n(t)$ is given by the observation process (7.3), wher f_n $[t,\ \theta(t)]$ can be considered approximated model to $n(t)$ and $\epsilon_n v_n(t)$ means modeling errors. In addition, it is assumed that ϵ_n is arbitrary real constant matrix and $v_n(t)\sim$ $N[O,\ R^{(n)}(t)]$ is white noise. $M_n\times M_n$ matrix $R^{(n)}(t)$ is symmetric positive definite and time continuous where $u_\theta(t)$ and $v_n(t)$ are uncorrelated. And $f_\theta[t,\theta(t)]$ as well as f_n $[t,\ \theta(t)]$ is time continuousnonlinear vector value functional with fitted size. Fur-thermore time continuous matrix $g_\theta(t)$ has also fitted size. Initial state N_θ-vector θ_0 is gaussian stochastic constant vector with mean $\bar{\theta}_0$ and covariance matrix $p_0{}^{(\theta)}$ where

$$\bar{\theta}_0 = E[\theta_0] \tag{8.1}$$

$$p_0{}^{(\theta)} = E[(\theta_0 - \bar{\theta}_0)\ (\theta_0 - \bar{\theta}_0)^T] \tag{8.2}$$

moreover $u_\theta(t)$ and $v_n(t)$ are uncorrelated to θ_0.

After all, the received signal N_n-vector $y(t)$ in Fig. 1 can be obtained. Before we set up $y(t)$, $z(t)$ is assumed to be as follows.

$$z(t) = \mathrm{H}[w(t),\ m(t)] = H_0[w(t),\ m(t)] + \epsilon_z v_z(t) \tag{9}$$

where H_0 means known functional and the term $\epsilon_z v_z(t)$ denotes approximated errors when H is approximated by H_0. Furthermore M_z-vector $v_z(t) \sim N[O,\ R^{(z)}(t)]$ and assumed white noise. And time continuous $M_z \times M_z$ symmetric matrix $R^{(z)}(t)$ is positive definite and ϵ_z is well fitted constant matrix. Thus $y(t)$ can be obtained now.

$$y(t) = z(t) + n(t) = H_0[w(t),\ m(t)] + f_n[t,\ \theta(t)] + v(t) \tag{10}$$

where

$$v(t) = \epsilon_z v_z(t) + \epsilon_n v_n(t) \tag{11}$$

Then $v_z(t)$ aed $v_n(t)$ are assumed independent each other. If we define now matrix $R(t)$, that is

$$R(t) = \epsilon_z R^{(z)}(t) \epsilon_z{}^T + \epsilon_n R^{(n)}(t) \epsilon_n{}^T \tag{12}$$

Then N_n-vector $v(t) \sim N[O,\ \mathrm{R}(t)]$ is also white noise, where $N_n \times N_n$ symmetric matrix $R(t)$ must be time continuous and positive definit.

We define here new state vector $x(t)$ and system noise $u(t)$:

$$x(t) = \begin{pmatrix} a(t) \\ \alpha(t) \\ \beta(t) \\ \theta(t) \end{pmatrix} \tag{13}$$

$$u(t) = \begin{bmatrix} u_a(t) \\ u_\beta(t) \\ u_\theta(t) \end{bmatrix} \tag{14}$$

Therefore we can obtain the following dynamical system from equations (1), (4), (5) and (7).

$$\dot{x}(t) = f[t,\ x(t)] + g(t)u(t) \tag{15.1}$$

$$\mathrm{x}(t_0) = x_0 \tag{15.2}$$

where initial state x_0 is

$$x_0 = \begin{pmatrix} a_0 \\ \alpha_0 \\ \beta_0 \\ \theta_0 \end{pmatrix} \tag{16}$$

Thus $x(t)$ is N-vector aud $u(t)$ is M-vector, where $N = N_a + N_\alpha + N_\beta + N_\theta$ and $M = M_a + M_\beta + M_\theta$. In equation (15.1), time continuous vector value functional $f[t,\ x(t)]$ and time continuous $N \times M$ matrix $g(t)$ can be expressed by the following equation.

$$f[t,\ x(t)] = \begin{pmatrix} f_a[t, a(t)] \\ F_\alpha(t)\,\alpha(t) + G_\alpha(t)\,a(t) \\ f_\beta[t,\ {}_\beta(t)] \\ f_\theta[t,\ \theta(t)] \end{pmatrix} \tag{17}$$

$$g(t)=\begin{pmatrix} g_a(t) & O & O \\ O & O & O \\ O & g_\beta(t) & O \\ O & O & g_\theta(t) \end{pmatrix} \tag{18}$$

In the preceding equation, symdol O shows zero matrix with well fitted size. Then $u(t)\sim N[O,Q(t)]$ is white noise, where time continuous $M\times M$ matrix $Q(t)$ is symmetric nonnegative definite and $Q(t)$ is defined as follows.

$$Q(t)=\begin{bmatrix} Q^{(a)}(t) & O & O \\ O & Q^{(\beta)}(t) & O \\ O & O & Q^{(\theta)}(t) \end{bmatrix} \tag{19}$$

Initial state N-vector $x_0\sim N[\bar{x}_0, p_0]$ is stochastic vector which is uncorrelated to both of $u(t)$ and $v(t)$. Furthermore, u(t) and $v(t)$ are independent each other. Then mean $\bar{x}_0$ of x_0 and covariance p$_0$ of x_0 are defined as follows.

$$\bar{x}_0=E[x_0]=\begin{pmatrix} \bar{a}_0 \\ \bar{\alpha}_0 \\ \bar{\beta}_0 \\ \bar{\theta}_0 \end{pmatrix} \tag{21}$$

$$p_0=E[(x_0-\bar{x}_0)(x_0-\bar{x}_0)^T] \tag{21}$$

where p_0 is generally nonnegative definite. And then from equation (10), $y(t)$ can be written as follows.

$$y(t)=h[t, x(t)]+v(t) \tag{22}$$

where

$$h[t, x(t)]=H_0[w(t), m(t)]+f_n[t, \theta(t)] \tag{23}$$

and $v(t)$ is given by equation (11). But equations (15.1) and (22) are mathematically fiction and as well known, they should be reformed by Itô stochastic differential equation[5], that is

$$dx(t)=f[t, x(t)]dt+g(t)dB^{(u)}(t) \tag{24}$$

$$dy^{(r)}(t)=h[t, x(t)]dt+dB^{(v)}(t) \tag{25}$$

where the following relations are supposed formally.

$$y(t)=\frac{dy^{(r)}(t)}{dt} \tag{26}$$

$$u(t)=\frac{dB^{(u)}(t)}{dt} \tag{27}$$

$$v(t)=\frac{dB^{(v)}(t)}{dt} \tag{28}$$

In the preceding relations (27) and (28), $B^{(u)}(t)$ and also $B^{(v)}(t)$ are Wiener processes with the following properties.

$$E[(B^{(u)}(t)-B^{(u)}(t_0)](B^{(u)}(s)-B^{(u)}(t_0)^T]$$

$$=\int_{t_0}^{min(t,\ s)} Q(\tau)d\tau \tag{29}$$

$$E[(B^{(v)}(t)-B^{(v)}(t_0))(B^{(v)}(s)-B^{(v)}(t_0))^T]$$

$$= \int_{t_0}^{min(t,\ s)} R(\tau)d\tau \tag{30}$$

where $B^{(u)}(t)$ and $B^{(v)}(t)$ are independent each other. Initial state x_0 is uncorrelated to both of $B^{(u)}(t)$ and $B^{(v)}(t)$. But it must be noted that the expressions of equations (15.1) and (22) give more intuitive view in mathematical treatment than the equations (24) and (25). Hence we adopt here (15.1) and (22).

3. Discussion

When we simulate the dynamical system (15.1) and the observed process (10), Itô stochastic differential equations (24) and (25) are practically or rigorously only applicable. Furthermore given observed value (received signal) in simulation is not $y(t)$ but only $y^{(r)}(t)$. Moreover in communication systems or transmission systems, we needs usually only one of elements, in other word, one of state variables. Approximation error terms $\epsilon_n v_n(t)$ in equation (7.3) and $\epsilon_z v_z(t)$ in equation (9) denote the observed noises and moreover the covariances of $\epsilon_n v_n(t)$ and $\epsilon_z v_z(t)$ must be practically unknown. Therefore, estimation (filtering) problems as well as control and identification problems become very hard. And we must note that the approximated terms $\epsilon_n v_n(t)$ and $\epsilon_z v_z(t)$ may be, generally or practically, functionals dependent on state variables. Therefore $\epsilon_n v_n(t)$ should be described dy $\epsilon_n[t,\ \theta(t)]v_n(t)$ and in the same reason $\epsilon_z v_z(t)$ can be written by $\epsilon_z[w(t),\ m(t)]v_z(t)$. Hence observed process can be expressed as follows.

$$y(t)=h[t,\ x(t)]+\epsilon(t,\ x(t))v(t) \tag{31}$$

or

$$dy^{(r)}(t)=h[t,\ x(t)]dt+\epsilon(t,\ x(t))dB^{(v)}(t) \tag{32}$$

where $\epsilon(t,\ x(t))v(t)$ corresponds to the extended formula to $v(t)$ in equatron (11). Then we may be required complicated mathematical treatment in estimation, control and identification problems.

4. Optical fiber transmission systems

For an illustrative example, we discuss the optical fiber transmission system as shown in Fig. 2. Fig. 2 shows a single mode optical transmission and furthermore

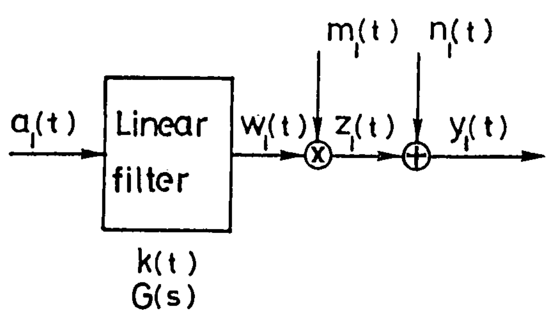

Fig. 2 An optical fiber transmission system model

transmitted signal $a_1(t)$ is given by direct modulation[2] where subscript 1 denotes the first element of state vector $a(t)$ and besides other state vectors or vector value functionals. Then we show system models as follows.

Transmitted signal:

For simplisity it is assumed that $a_1(t)$ is single impulse with the dimension $N_a=1$:

$$a_1(t)=A\delta(t-t_1) \tag{33. 1}$$

where A, $t_1>O$ are arbitrary constant. But equation (33. 1) is mathematical idea, so that we assume $a_1(t)$ as follows.

$$a_1(t)=A/\sqrt{2\pi}\sigma\ exp[-(t-t_1)^2/2\sigma^2]\ (\sigma>O \text{ is sufficiently small}) \tag{33. 2}$$

This is correct from the following relation:

$$\delta(t-t_1)=\lim_{\sigma\to 0}1/\sqrt{2\pi}\sigma\ exp[-(t-t_1)^2/2\sigma^2] \tag{33. 3}$$

Hence wa can obtain the dynamical system

$$\dot{a}_1(t)=-d(t-t_1)a_1(t) \tag{34. 1}$$

where

$$\mathrm{d}=1/\sigma^2$$

Transmission line:

We suppose that the transmission line has approximately band pass filter cheracteristic and its transfer function $G(s)$(it's impulse response is denoted by $K(t)$) is assumed as follows.

$$G(s)=\frac{2\epsilon s}{s^2+2\epsilon s+\omega_0^2} \tag{35}$$

where ϵ, $\omega_0>0$ and s is complex number. From equation (35), we can obtain the following dynamical system.

$$\dot{\alpha}(t)=F_a\alpha(t)+G_a(t)a_1(t) \tag{36. 1}$$

$$w_1(t)=\alpha_1(t) \tag{36. 2}$$

In this case $N_\alpha=1$, $N_w=1$. Where

$$\alpha(t)=\begin{bmatrix}\alpha_1(t)\\ \alpha_2(t)\end{bmatrix} \tag{36. 3}$$

$$F_a=\begin{bmatrix}0 & 1\\ -\omega_0^2 & -2\epsilon\end{bmatrix} \tag{36. 4}$$

$$G(t)=\begin{bmatrix}0\\ 2\epsilon d\,(t_1-t)\end{bmatrix} \tag{36. 5}$$

Nonliear coupling noise:

We supposed $m_1(t)$ as follows.

$$m_1(t)=m_1^{(1)}(t)+m_1^{(0)} \tag{37}$$

where

$$E[m_1(t)]=E[m_1^{(0)}] \tag{38}$$

Then

$$E[m_1^{(1)}(t)]=0 \tag{39}$$

Obviously, it must be noted that the expectation of $m_1(t)$, that is, $E[\mathrm{m}_1^{(0)}]$, has very important role in the transmission lines. Now we define the following dynamical system.

$$\dot{\beta}_1(t)=\beta_2(t) \tag{40}$$

$$\dot{\beta}_2(t)=-\beta_c\beta_1(t)-\alpha_c\beta_2(t)+\gamma u_1^{(\beta)}(t) \tag{41}$$

$$\beta_c,\ \alpha_c,\ \gamma>0$$

where

$$\beta_1(t)=m_1^{(1)}(t) \tag{42}$$

and $u_1^{(\beta)}(t)\sim N[0,\ q^{(\beta)}(t)]$ is white noise.

If

$$\beta_3(t)=m_1^{(0)} \tag{43}$$

then

$$\dot{\beta}_3(t)=0 \tag{44}$$

is true. Therefore, from the set of equations (40), (41), (44) and (38), the following dynamical systm is correct.

$$\dot{\beta}(t)=F_\beta\ \beta(t)+G_\beta u_1^{(\beta)}(t) \tag{45.1}$$

$$m_1(t)=\beta_1(t)+\beta_3(t) \tag{45.2}$$

$$\beta(t)=\begin{bmatrix}\beta_1(t)\\ \beta_2(t)\\ \beta_3(t)\end{bmatrix} \tag{45.2}$$

where $N_\beta=3$, $N_m=M_\beta=1$ and

$$F=\begin{bmatrix}0 & 1 & 0\\ -\beta_c & -\alpha_c & 0\\ 0 & 0 & 0\end{bmatrix} \tag{45.4}$$

$$G=\begin{bmatrix}0\\ \gamma\\ 0\end{bmatrix} \tag{45.5}$$

Additive noise:

In practical meaning, additive noise $n_1(t)$ in Fig.2 may not be gaussian, because it is mathematical idea. So, generally we should be assume that $n_1(t)$ is non-gaussian and also non-white noise. Under these conditions, we first give the following Langevin equation.

$$\dot{\theta}_1(t)=-f_\theta\theta_1(t)+G_\theta u_1^{(\theta)}(t) \tag{46}$$

where $u_1^{(\theta)}(t)\sim N[0,\ q^{(\theta)}(t)]$ is white noise. Then we define $n_1(t)$ as follows.

$$n_1(t)=\lambda\theta_1^2(t)+v_1(t) \tag{47}$$

where also $v_1(t)\sim N[0,\ r^{(v)}(t)]$ is white noise and $N_\theta=N_n=M_n=M_v=1$

Observed value (Received signal):

In Fig. 2, the output signal $z_1(t)$ of memoryless nonlinear transducer is the product of $w_1(t)$ and $m_1(t)$, however it is assumed that approximated term $\epsilon_z v_z(t)$ in equaton (9) is zero, in other words, transducer has known structure.

$$z_1(t)=w_1(t)m_1(t)=\alpha_1(t)\beta_1(t)+\alpha_1(t)\beta_3(t) \tag{48}$$

This equation can be soon obtained from equation (36.2), (45.2). Concequently, we can define observed process or received signal $y_1(t)$ as follows.

$$y_1(t)=z_1(t)+n_1(t)=\alpha_1(t)\beta_1(t)+\alpha_1(t)\beta_3(t)+\lambda\theta_1^2(t)+v_1(t) \tag{49}$$

Now we define new state vector

$$x(t)=\begin{pmatrix} x_1(t)\\ x_2(t)\\ x_3(t)\\ x_4(t)\\ x_5(t)\\ x_6(t)\\ x_7(t)\end{pmatrix}=\begin{pmatrix} a_1(t)\\ \alpha_1(t)\\ \alpha_2(t)\\ \beta_1(t)\\ \beta_2(t)\\ \beta_3(t)\\ \theta_1(t)\end{pmatrix} \tag{50}$$

As a result, we can obtain the following dynamical system of $x(t)$

$$x(t)=F(t)x(t)+g(t)u(t) \tag{5}$$

where

$$u(t)=\begin{bmatrix} u_1^{(\beta)}(t)\\ u_1^{(\theta)}(t)\end{bmatrix} \tag{52}$$

$$F(t)=\begin{pmatrix} -d(t-t_1) & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 1 & 0 & 0 & 0 & 0\\ 2\epsilon d(t_1-t) & -\omega_0^2 & -2\epsilon & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & -\beta_c & -\alpha_c & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & -f_\theta \end{pmatrix} \tag{53}$$

$$g(t)=\begin{pmatrix} 0 & 0\\ 0 & 0\\ 0 & 0\\ 0 & 0\\ \gamma & 0\\ 0 & 0\\ 0 & G_\theta \end{pmatrix} \tag{54}$$

Naturally, the initial condition of $x(t)$ can be defined as follows.

$$x(t_0)=\begin{pmatrix} x_1(t_0)\\ x_2(t_0)\\ x_3(t_0)\\ x_4(t_0)\\ x_5(t_0)\\ x_6(t_0)\\ x_7(t_0)\end{pmatrix}=\begin{pmatrix} a_1(t_0)\\ \alpha_1(t_0)\\ \alpha_2(t_0)\\ \beta_1(t_0)\\ \beta_2(t_0)\\ \beta_3(t_0)\\ \theta_1(t_0)\end{pmatrix} \tag{55}$$

Finally observed process can be written as follows.

$$y_1(t)=h_1[t,\ x(t)]+v_1(t) \tag{56}$$

where

$$h_1[t,x(t)]=x_2(t)x_4(t)+x_2(t)x_6(t)+\lambda x_7^2(t) \tag{57}$$

and observed noise $v_1(t)$ corresponds to approximated term $\epsilon_n v_n(t)$ in equation (11). Moreover the dimension is $N=7$, $N_n=1$. Clearly observed process is nonlinear.

5. Conclusion

It was discussed how to construct the dynamical systems to the general transmission systems with nonlinear coupling noises. Furthermore, for an illustrative example, the dynamical system to typical optical fiber transmission system was obtained in chapter 4. In fact, wa must again note that once given the dynamical systems, it can be applicated easy to estimation, control and identification problems. Hence the results of this paper are very important and useful.

Obiviously this paper was discussed of only continuous systems but it is not essential. In reality, discrete systems can be also obtained in the same way.

However, first, it must be thought that the dimensions of dynamical systems increase as soon as the dimension of dynamical system to the transmitted signals. Then much complicated treatment must be required. Secondly, it must be noted that as a rule, system noises and also observed noises have unknown covariance matrices and generally must be dependent on the state of systems. In their situation, mathematical treatment in applications becomes very complicated.

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